1 Predicate Logic Symbolization

- **innovation of predicate logic**: analysis of simple statements into two parts: the subject and the predicate.
- **E.g. 1**:
  'John is a giant'.
  subject = 'John'
  predicate = '…is a giant'
  Predicate logic symbolization:
  j = 'John', G = 'is a giant'
  Gj
- In predicate logic, the subject identified is called an **individual constant**
  predicate is called the **symbolic predicate**
- Note: Predicate logic also inherits all the logical connectives from propositional logic ($\sim, \bullet, \vee, \supset, \equiv$).

English Statement	Propositional Logic	Predicate Logic
Bermuda is a country.	B	Cb
Bermuda is an island and a country.	I • C	Ib • Cb
If Bermuda is a country then Jamaica is a country.	B ⊃ J	Cb ⊃ Cj
JFK is on high alert just in case O'Hare is.	J ≡ O	Hj ≡ Oj

Quantification

- The breaking down of simple statements into two parts allows for a novel treatment of English statements that include 'all', 'everything', 'no', 'none', 'some'
- New symbols:
  $\forall$, the **universal quantifier**, read 'for all'
  $\exists$, the **existential quantifier**, read 'some' or 'at least one'

English Statement	Re-wording	PL Symbolization
Tigers are animals.	All tigers are animals	(x)(Tx ⊃ Ax)
No tigers are canine.	All tigers are not canine	(x)(Tx ⊃∼ Cx)
Everything is round.	For all x, x is round.	(x)(Rx)

English Statement	Re-wording	PL Symbolization
Some tigers are albino.	At least one tiger is albino.	$(\exists x)(Tx \bullet Ax)$
Some tigers are not in captivity	At least one tiger is not in captivity	$(\exists x)(Tx\bullet \sim Cx)$

2 Predicate Logic - Rules of Inference

Some Terminology

- **quantifiers**: $(x) \ldots$, $(\exists x) \ldots$
- **constants**: typically labeled $a, b, \ldots, t$
- **variables**: typically labeled: $x, y, z, x_1, \ldots, y_1, \ldots$
  - *bound variables* - variables in statements bound by quantifiers
  - *free variables* - variables in statement functions not bound by quantifiers
- **statement function** - the expression that remains when a quantifier is removed from a statement

- **E.g. 1**: (x)(Sx ⊃ Tx)
  statement (universal generalization)
  x is a bound variable
  no free variable
- **E.g. 2**: (∃y)(Sy • Ty)
  statement (existential generalization)
  y is a bound variable
  no free variables
- **E.g. 3**: Fy ⊃ Gy
  statement function
  y is a free variable
  no bound variables

Rule of Inference	Rule Form	Conditions on Proper Use
Universal Instantiation (UI):	(x)Fx // Fa	None
	(x)Fx // Fy	None

- **E.g. 1**: Universal Instantiation
  1. Everything is physical.
  2. So, Mars is physical.

  Px — x is physical
  m — Mars

1	$(x)Px$	
2	Pm	1, UI

- **E.g. 2**: Universal Instantiation
  1. All economists are social scientists.
  2. Paul Krugman is an economist.
  3. Therefore, Paul Kraugman is a social scientist.

  Ex — x is an economist
  Sx — x is a social scientist
  p — Paul Krugman

1	$(x)(Ex \supset Sx)$	Premise
2	Ep	Premise
3	$Ep \supset Sp$	1 UI
4	Sp	2, 3 MP

- **E.g. 3**: Universal Instantiation

1	$(x)(Hx \supset Gx)$	
2	$Hy \supset Gy$	UI

Rule of Inference	Rule Form	Conditions on Proper Use
Existential Generalization (EG):	Fa // (∃ x)Fx	None
	Fy // (∃ x)Fx	None

- **E.g. 1**: Existential Generalization

1	Lp	
2	$(\exists x)(Lx)$	1 EG

- **E.g. 2**: Existential Generalization

1	$Lj \bullet Hg$	
2	$(\exists x)(Lj \bullet Hx)$	1 EG
3	Lj	1 Simp
4	$(\exists x)(Lx)$	3, EG

Rule of Inference	**Rule Form**	**Conditions on Proper Use**
Universal Instantiation (UI):	(x)Fx // Fa	None
	(x)Fx // Fy	None
Universal Generalization (UG):	Fy // (x)Fx	i) not allowed: Fa // (x)Fx; ii) cannot use UG on a free variable in the first line of sub-derivation
Existential Generalization (EG):	Fa // (∃ x)Fx	None
	Fy // (∃ x)Fx	None
Existential Instantiation (∃I):	(∃ x)Fx // Fa	i) the existential name α must be a new name that does not appear in any previous line (including the conclusion line) ii) not allowed: (∃ x)Fx // Fy

- **E.g. 1: Universal Generalization**
  Note: we cannot infer a universal statement from a particular: Fa // (x)Fx is incorrect use of rule.

1	Ly	
2	$(x)(Lx)$	1 UG

- **E.g. 2: Universal Generalization**

1	$Fy \supset Gy$	Premise
2	$Fy \supset Gx$	Premise
3	$(y)(Fy \supset Gy)$	1 UG
4	$(z)(Fz \supset Gx)$	2 UG

- **E.g. 3:** $(x)(Hx \supset Ix)$, $(x)(Ix \supset Hx)$ // $(x)(Hx \equiv Ix)$
- **E.g. 4: Universal Instantiation (w/ statement functions)**

1	$(x)(Px \supset Dx)$	Premise
2	$(x)(Dx \supset Cx)$	Premise
3	$Py \supset Dy$	1 UI
4	$Dy \supset Cy$	2, UI
5	$Py \supset Cy$	3, 4 HS
6	$(x)(Px \supset Cx)$	5, UG

Rule of Inference	**Rule Form**	**Conditions on Proper Use**
Universal Instantiation (UI):	(x)Fx // Fa	None
	(x)Fx // Fy	None
Universal Generalization (UG):	Fy // (x)Fx	i) not allowed: Fa // (x)Fx
Existential Generalization (EG):	Fa // (∃ x)Fx	None
	Fy // (∃ x)Fx	None

- **E.g. 4: Existential Generalization**

1	$(x)[(Ax \vee Bx) \supset Cx]$	Premise
2	$(\exists x)Ax$	Premise
3	Am	2, EI
4	$(Am \vee Bm) \supset Cm$	1 UI
5	$Am \vee Bm$	3, Add
6	Cm	4, 5 MP
7	$(\exists x)Cx$	6, EG

- **E.g. 5: Existential Generalization**

1	$(\exists x)Kx \supset (x)(Lx \supset Mx)$	Premise
2	$Kc \bullet Lc$	Premise
3	Kc	2, Simp
4	$(\exists x)Kx$	3, EG
5	$(x)(Lx \supset Mx)$	1, 4 MP
6	$Lc \supset Mc$	5, UI
7	$Lc \bullet Kc$	2, Comm
8	Lc	7, Simp
9	Mc	6, 8 MP

- **E.g. 2**: $(x)(Px \supset Qx) \supset (\exists x)(Rx \bullet Sx)$ // $(\exists x)Sx$
- **Ex 8.2.6**: $(x)[Jx \supset (Kx \bullet Lx)]$, $(\exists y)(\sim Ky)$ // $(\exists z) \sim Jz$

3 Change of Quantifier Rule

$$
\begin{array}{ccc}
(x)Fx & :: & \sim(\exists x)\sim Fx \\
\sim(x)Fx & :: & (\exists x)\sim Fx \\
(\exists x)Fx & :: & \sim(x)\sim Fx \\
\sim(\exists x)Fx & :: & (x)\sim Fx
\end{array}
$$

- **E.g. 1**: $(\exists x)(Hx \bullet Gx) \supset (x)Ix, \sim Im // (x)(Hx \supset\sim Gx)$

1	$(\exists x)(Hx \bullet Gx) \supset (x)Ix$	Premise
2	$\sim Im$	Premise
3	$(\exists x)\sim Ix$	2 EG
4	$\sim(x)Ix$	3 CQ
5	$\sim(\exists x)(Hx \bullet Gx)$	1, 4 MT
6	$(x)\sim(Hx \bullet Gx)$	5 CQ
7	$(x)(\sim Hx \vee \sim Gx)$	6, DM
8	$(x)(Hx \supset\sim Gx)$	7 Impl

- **Ex. 8.3.I.11**: $\sim (\exists x)(Ax \bullet \sim Bx), \sim (\exists x)(Ax \bullet \sim Cx)$ // $(x)[Ax \supset (Bx \bullet Cx)]$

1	$\sim (\exists x)(Ax \bullet \sim Bx)$	Premise
2	$\sim (\exists x)(Ax \bullet \sim Cx)$	Premise
3	$(x) \sim (Ax \bullet \sim Bx)$	1 CQ
4	$(x) \sim (Ax \bullet \sim Cx)$	2 CQ
5	Ay	ACP
6	$\sim (Ay \bullet \sim By)$	3, UI
7	$\sim (Ay \bullet \sim Cy)$	4, UI
8	$\ldots$	
9	$By \bullet Cy$	Conj
10	$Ay \supset (By \bullet Cy)$	5-9 Cp
11	$(x)[Ax \supset (Bx \bullet Cx)]$	10 UG

- **Ex. 8.3.I.6**:

1	$(\exists x) \sim Ax \supset (x)(Bx \supset Cx)$	Premise
2	$\sim (x)(Ax \vee Cx)$	Premise
3	$(\exists x) \sim (Ax \vee Cx)$	2 CQ
4	$\sim (Ai \vee Ci)$	3 EI
5	$\sim Ai \bullet \sim Ci$	4 Dem
6	$\sim Ai$	5, Simp
7	$(\exists x) \sim Ax$	6 EG
8	$(x)(Bx \supset Cx)$	1, 7 MP
9	$Bi \supset Ci$	8 UI
10	$\sim Ci \bullet \sim Ai$	5 Comm
11	$\sim Ci$	10 Simp
12	$\sim Bi$	9, 11 MT
13	$(\exists x) \sim Bx$	12 EG
14	$\sim (x)Bx$	13 CQ