1 | Symbols and Translation (Ch 6.1)

- Our aim for the rest of the course is to study valid (deductive) arguments in symbolic systems. In this section, (Ch. 6-7) we look at the symbolic system of propositional logic.
- Once informal arguments in English are translated into the symbolic language of propositional logic (Ch. 6.1), they and their properties can be studied with greater precision (Ch. 6.2-6.5).

Dfn **1.1.** *Operators: and, or, if, if and only if*
Dfn **1.2.** *Simple Statement: a statement with no operators.*
Dfn **1.3.** *Compound Statement: a statement with at least one operator.*

Operator	Name	Logical Function	Some English synonyms
$\sim$	tilde	negation	not, it is not the case that
$\bullet$	dot	conjunction	and, also, moreover, but
$\vee$	wedge	disjunction	or, unless
$\supset$	horseshoe	implication	if ... then ..., only if,
$\equiv$	triple bar	equivalence	if and only if, just in case

Simple Statements

- John is a mailman.
- Rolex makes computers.
- *Symbolization*:
  J = "John is a mailman"
  R = "Rolex makes computers."

Negation

- Rolex does **not** make computers.
- **It is not the case that** Rolex makes computers.
- **It is false that** Rolex makes computers.
- *Symbolization*:
  R = "Rolex makes computers."
  $\sim R$

Conjunctive Statements

- Tiffany sells jewelry **and** Gucci sells cologne.
- Tiffany sells jewelry, **but** Gucci sells cologne.
- *Symbolization*:
  T = "Tiffany sells jewelry."
  G = "Gucci sells cologne."
  $T \bullet G$
- Tiffany and Ben Bridges sell jewelry.
- *Symbolization*:
  T = "Tiffany sells jewelry."
  B = "Ben Bridges sells jewelry."
  $T \bullet B$

Disjunctive Statements

- Aspen allows snowboards **or** skis.
- **Either** Aspen allows snowboards **or** skis.
- Aspen allows snowboards **unless** they allow skis.
- **Unless** Aspen allows snowboards, they allow skis.
- *Symbolization*:
  A = "Aspen allows snowboards"
  S = "Aspen allows skis"
  $A \vee S$

Conditional Statements

- **If** Purdue raises tuition, **then** so does Notre Dame
- Notre Dame raises tuition **if** Purdue does
- Purdue raises tuition **only if** Notre Dame does
- Notre Dame raises tuition **provided that** Purdue raises tuition
- *Symbolization*:
  P = "Purdue raises tuition"
  N = "Notre Dame raises tuition"
  $P \supset N$

Biconditional Statements

- JFK tightens security **if and only if** O'Hare tightens security.
- JFK tightens security **just in case** O'Hare tightens security.
- JFK's tightening security **is necessary and sufficient for** O'Hare doing so.
- *Symbolization*:
  J = "JFK tightens security"
  O = "O'Hare tightens security"
  $J \equiv O$

Dfn **1.4.** *Main Operator: the operator that has as its scope everything else in a compound statement.*

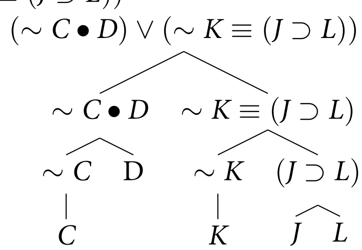
Dfn **1.5.** *Well-formed formula (wff): a syntactically correct statement of the symbolic language*

The Structure of Wffs

- Two Questions:
  i) Is the statement a well-formed formula (wff)?
  ii) If so, what is the main operator?
- **Method**: ** This method is not covered in the textbook.**
  - **wff**: If a tree can be constructed for a statement, then it is a wff, and its top node is the main operator.
  - **not a wff**: If a tree cannot be constructed for a statement, it is not a wff.
- Note: Brackets are often omitted around negation signs
  - $A \bullet (\sim B \vee C)$ is read as $A \bullet ((\sim B) \vee C)$ and not $A \bullet (\sim (B \vee C))$
  - $B \supset\sim (\sim\sim A \equiv B)$ is read as $B \supset\sim ((\sim (\sim A)) \equiv B)$

Examples

- **E.g. 1**: $(\sim C \bullet D) \vee (\sim K \equiv (J \supset L))$

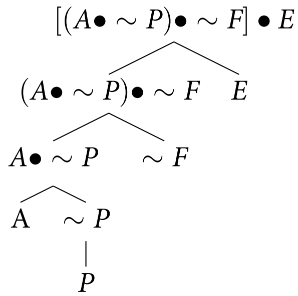


  the statement is a wff; main operator: $\vee$

- **E.g. 2**: $M(N \supset Q) \vee (\sim C \bullet D)$ [Exercise. 6.1.III.8]
  this tree can't be constructed. So, this string of symbols is not a wff.

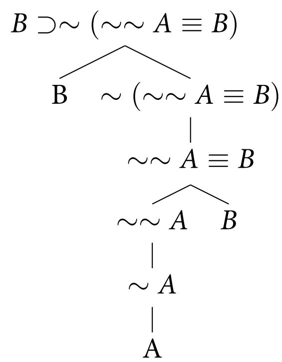
- **E.g. 3**: $(N \supset \bullet Q) \equiv (C \vee D)$
  this tree can't be constructed. So, this string of symbols is not a wff.

- **E.g. 4**: $[(A\bullet \sim P)\bullet \sim F] \bullet E$



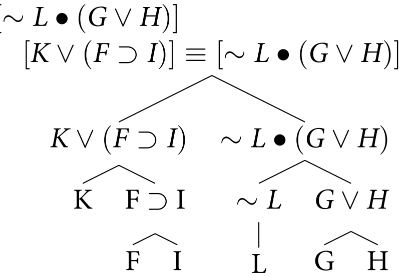
  the statement is a wff; main operator: 3rd $\bullet$

- **E.g. 5**: $B \supset \sim (\sim\sim A \equiv B)$



  the statement is a wff; main operator: $\supset$

- **E.g. 6**: $[K \vee (F \supset I)] \equiv [\sim L \bullet (G \vee H)]$



  the statement is a wff; main operator: $\equiv$

- **E.g. 7**: $[K \vee (S\bullet \sim\sim T)] \supset [\sim F \vee (M \bullet O)]$

Truth-Functions (Ch. 6.2)

Negation ($\sim$)

P	$\sim P$
T	F
F	T

- **E.g. 1**: It is not the case that Medea is evil.
  M = Medea is evil
  $\sim M$
- **E.g. 2**: Medea didn't spare her children.
  M = Medea spared her children
  $\sim M$
- **E.g. 3**: Jason never was a good husband.
  J = Jason was a good husband
  $\sim J$

Conjunction Statements

P	Q	$P \bullet Q$
T	T	T
T	F	F
F	T	F
F	F	F

- **E.g. 1**: Jason is Greek and Medea is Greek.
  J = "Jason is Greek", M = "Medea is Greek"
  $J \bullet M$
- **E.g. 2**: Jason is rich; however, Medea is not.
  J = "Jason is rich", M = "Medea is rich"
  $J \bullet \sim M$

Disjunction Statements

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

- **E.g. 1**: Either Shakespeare or Euripides is a playwright. $S \vee E$
- **E.g. 2**: Euripides is a misogynist or a feminist. $(M \vee F) \bullet \sim (M \bullet F)$
- **E.g. 3**: The meal comes with either soup or salad. $(P \vee S) \bullet \sim (P \bullet S)$

  Note: the closest translation of this statement is not $P \vee S$, since disjunctions are true when both disjuncts are true (first row), but normally we mean that the meal comes with one item, soup or salad, but not both. Compare the **e.g. 1**: in this case, Shakespeare is a playwright and Euripides is a playwright, and the statement is true.

Conditional Statements

P	Q	$P \supset Q$
T	T	T
T	F	F
F	T	T
F	F	T

- **E.g. 1**: If Purdue raises tuition, then so does Notre Dame. $P \supset N$
- **E.g. 2**: Purdue raises tuition provided that Notre Dame does. $N \supset P$

Biconditional Statements

P	Q	$P \equiv Q$
T	T	T
T	F	F
F	T	F
F	F	T

- **E.g. 1**: JFK tightens security just in case O'Hare does. $J \equiv O$
- **E.g. 2**: JFK tightens security if and only if O'Hare does $J \equiv O$
- Note: $J \equiv O$ is identical to $(J \supset O) \bullet (O \supset J)$

Differences between Ordinary Language and Logical Connectives

- **E.g. 1**: She got married and had a baby.
  She had a baby and got married.
  Symbolization: $M \bullet B$, or $B \bullet M$ (they are equivalent).
- The temporal aspects is lost in the translation

- **E.g. 2**: (A) If Shakespeare wrote Hamlet, then the sun rises in the east.
  (B) If Shakespeare wrote Hamlet, then the sun rises in the west.
  Symbolization:
  (A) $S \supset E$ — true
  (B) $S \supset W$ — false

- **E.g. 3**: If Shakespeare wrote Hamley then the sun rises in the East (true)
  If ice is lighter than water, then ice floats in water.
  Note: these are both conditionals, but one has an link between statements and the other does not. Both are true.

Truth-Tables for Logical Operators

P	$\sim P$
T	F
F	T

P	Q	$P \bullet Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	$P \supset Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	$P \equiv Q$
T	T	T
T	F	F
F	T	F
F	F	T

Evaluating Truth Values of Compound Statements (Ch. 6.2)

Let A, B, C be true; Let Y, Z be false.

- **E.g. 1** $\sim C \vee B$

$\sim$	C	$\vee$	B
F	T	**T**	T

- **Ex. 6.2.III.12**: $A \supset\sim (Z\vee \sim Y)$

A	$\supset$	$\sim$	$(Z$	$\vee$	$\sim$	$Y)$
T	**F**	F	F	T	T	F

Constructing Truth Tables of Compound Statements (Ch. 6.3)

Ths **1.6.** *The relationship between the number of simple statements (n) and number of rows in the truth table (L) is* $L = 2^n$

- **Ex. 6.3.I.5**: $(\sim K \supset H) \equiv\sim (H \vee K)$

K	H	$(\sim$	K	$\supset$	$H)$	$\equiv$	$\sim$	$(H$	$\vee$	$K)$
T	T	F	T	T	T	**F**	F	T	T	T
T	F	F	T	T	F	**F**	F	F	T	T
F	T	T	F	T	T	**F**	F	T	T	F
F	F	T	F	F	F	**F**	T	F	F	F

- **Ex. 6.3.I.5**: $(\sim K \supset H) \equiv\sim (H \vee K)$

K	H	$\sim$	K	$\supset$	H	$\equiv$	$\sim$	H	$\vee$	K
T	T	F	T	T	T	**T**	F	T	T	T
T	F	F	T	T	F	**T**	T	F	T	T
F	T	T	F	T	T	**T**	F	T	F	F
F	F	T	F	F	F	**F**	T	F	T	F

- **Ex. 6.3.I.11**: $[(Q \supset P) \bullet (\sim Q \supset R)] \bullet \sim (P \vee R)]$

P	Q	R	$[(Q$	$\supset$	$P)$	$\bullet$	$(\sim$	Q	$\supset$	$R)]$	$\bullet$	$\sim$	$(P$	$\vee$	$R)$
T	T	T	T	T	T	T	F	T	T	T	**F**	F	T	T	T
T	T	F	T	T	T	T	F	T	T	F	**F**	F	T	T	F
T	F	T	F	T	T	T	T	F	T	T	**F**	F	T	T	T
T	F	F	F	T	T	F	T	F	F	F	**F**	F	T	T	F
F	T	T	T	F	F	F	F	T	T	T	**F**	F	F	T	T
F	T	F	T	F	F	F	F	T	T	F	**F**	T	F	F	F
F	F	T	F	T	F	T	T	F	T	T	**F**	F	F	T	T
F	F	F	F	T	F	F	T	F	F	F	**F**	T	F	F	F

2 Truth Tables for Propositions (Ch. 6.3)

- a **single statement** is
  - **logically true** (Tautologous) just in case its truth value under the main operator in a truth table is always true
  - **logically false** (Self-Contradictory) just in case its truth value under the main operator in a truth table is always false
  - **contingent** just in case its truth value under the main operator in a truth table is sometimes true and sometimes false
- **E.g. 1**: $(A \supset\sim (B \bullet A)) \vee \sim\sim B$ (logically true)

A	B	$(A$	$\supset$	$\sim$	$(B$	$\bullet$	$A))$	$\vee$	$\sim$	$\sim$	$B)$
T	T	T	F	F	T	T	T	**T**	T	F	T
T	F	T	T	T	F	F	T	**T**	F	T	F
F	T	F	T	T	T	F	F	**T**	T	F	T
F	F	F	T	T	F	F	F	**T**	F	T	F

- **E.g. 2**: If the Mona Lisa is beautiful then the Mona Lisa is beautiful.
- **E.g. 3**: It is neither raining nor not raining.

- **Two statements** are
  - **logically equivalent statements** just in case they have *the same truth value in each row* of truth table under the main operator
  - **contradictory statements** just in case they have *opposite truth values in each row* of truth table under the main operator

- **Two or more statements** are
  - **consistent** just in case there is at least one row in the truth table in which all statements are true
  - **inconsistent** just in case there is no row in the truth table in which all statements are true
- **E.g. 1**: $\sim K \supset L, K \supset\sim L$ (These statements are **logically consistent**)

K	L	$\sim$	$(K \supset L)$	K	$\supset$	$\sim$	L
T	T	F	T		F	F	
T	F	T	F		T	T	
F	T	F	T		T	F	
F	F	F	T		T	T	

- **E.g. 2**: $W \equiv (B \bullet T), W \bullet (T \supset\sim B)$ inconsistent
- **E.g. 3**: $H \bullet (K \vee J), (J \bullet H) \vee (H \bullet K)$ equivalent
- **E.g. 4**: $(G \bullet (E \vee P), (\sim (G \bullet E) \bullet \sim (G \bullet P)$ consistent

3 Truth Tables for Arguments (6.4)

- How to tell that an **argument** is
  - **logically invalid**: there is at least one truth-table row with *all* true premises and a false conclusion
  - **logically valid**: there is no truth-table row with *all* true premises and a false conclusion
- **E.g. 1**: Logically valid
  1. $O \supset \sim Q$
  2. $\sim Q \supset B$

     —————-
  3. $O \supset B$

O	Q	B	O	$\supset$	$\sim$	Q	$\sim$	Q	$\supset$	B	$O \supset B$
T	T	T		F	F		F		T		T
T	T	F		F	F		F		T		F
T	F	T		T	T		T		T		T
T	F	F		T	T		T		F		F
F	T	T		T	F		F		T		T
F	T	F		T	F		F		T		T
F	F	T		T	T		T		T		T
F	F	F		T	T		T		F		T

- **E.g. 2**:
  The Yukon Dodgers basketball team is the best team in the country. Therefore, either it is raining or it is not.
  1. Y

     ——-
  2. $R \vee \sim R$

Y	R	Y	$\sim$	(R	$\vee$	$\sim$	R)
T	T	**T**	T	T	**T**	F	T
T	F	**T**	F	F	**T**	T	F
F	T	**F**	T	T	**T**	F	T
F	F	**F**	T	F	**T**	T	F

- **E.g. 4**: $O \supset \sim Q, \sim Q \supset B$, so, $O \supset B$
- **E.g. 5**: $E \supset (F \bullet G), F \supset (G \supset H)$, so $E \supset H$
- **E.g. 6**: $L \supset M, M \supset N, N \supset L$, so $L \vee N$

4 Significance of Propositional Logic

Benefits of Propositional Logic

- **Precision**: Precise symbolization eliminates ambiguity of ordinary language
- **Mechanical Computation**: Validity, Logical Truth, Logical Falsity, Consistency can be determined by a mechanical procedure

Limitation of Propositional Logic

- Here is a deductively valid argument according to Ch.1 dfn
  1. All men are mortal.
  2. Socrates is a man.

     ———

  3. So, Socrates is mortal.

  Symbolization in propositional logic:
  1. M
  2. S

     ———

  3. T

  But, the truth table of this argument tells us that there are rows with true premises and a false conclusion, so in propositional logic this argument is **logically invalid**.

- That propositional logic yields the wrong result here calls for an improvement in logic. Predicate logic, which we will look at in ch. 8, is an example of such an improvement.
- Here are some more arguments that are deductive valid according to definition of validity from Ch.1. but they are invalid according to definition of logical validity in propositional logic

  1. All men are mortal. Socrates is a man. So, Socrates is mortal.
  2. Jack knows that it is cold. So, Jack believes it is cold.
  3. Jack is a bachelor. Therefore, Jack is male.
  4. Axioms of Euclidean Geometry are true, so $a^2 + b^2 = c^2$ (Pythagorean Theorem)