1 Recursive Relations (§7.1)

- A set of, say, natural numbers is **effectively decidable** if there is an effective procedure that, applied to a natural number, in a finite amount of time gives the correct answer to the question whether it belongs to the set
- Thus, representing the answer 'yes' by 1 and the answer 'no' by 0, a set is **effectively decidable** if and only if its characteristic function is effectively computable, where the characteristic function is the function that takes the value 1 for numbers in the set, and the value 0 for numbers not in the set.
- A set is called **recursively decidable**, or simply recursive for short, if its characteristic function is recursive, and is called primitive recursive if its characteristic function is primitive recursive. Since recursive functions are effectively computable, recursive sets are effectively decidable. Church's thesis, according to which all effectively computable functions are recursive, implies that all effectively decidable sets are recursive.
- a two-place relation R among natural numbers will be simply a set of ordered pairs of natural numbers, and we write Rxy — or $R(x, y)$ if punctuation seems needed for the sake of readability — interchangeably with $(x, y) \in R$ to indicate that the relation R holds of x and y, which is to say, that the pair (x, y) belongs to R
- a k-place relation is a set of ordered k-tuples.
- The **characteristic function of a k-place relation** is the k-argument function that takes the value 1 for a k-tuple if the relation holds of that k-tuple, and the value 0 if it does not; and a relation is effectively decidable if its characteristic function is effectively computable, and is (primitive) recursive if its characteristic function is (primitive) recursive.

process for obtaining new recursive functions from old: what follows is actually a pair of propositions, one about primitive recursive functions, the other about recursive functions (according as one reads the proposition with or without the bracketed word 'primitive').
(Exm.1.1) (Identity and order). The identity relation, which holds if and only if x = y, is primitive recursive, since a little thought shows its characteristic function is 1 – (sg(x .– y) + sg(y .– x)). The strict less-than order relation, which holds if and only if x < y, is primitive recursive, since its characteristic function is sg(y .–x).
(Prop.1.2) (Definition by cases). Suppose that f is the function defined in the following form: f (x, y) = g1(x, y) if C1(x, y) ……gn(x, y) if Cn(x, y) where C1, …, Cn are (primitive) recursive relations that are mutually exclusive, meaning that for no x, y do more than one of them hold, and collectively exhaustive, meaning that for any x, y at least one of them holds, and where g1, …, gn are (primitive) recursive total functions. Then f is (primitive) recursive.

2 Recursive Relations (§7.2)