1 | Syntax

- **Propositional Connectives**: , , , ,
- **Atomic Sentences**: $p, p_1, p_2, \ldots, q, q_1, \ldots$ (an atomic sentence)

  Dfnφ is a sentence iff

  1. φ is an atomic sentence
  2. If φ and ψ are sentences, then so are (φ), $(\varphi\psi)$, $(\varphi\psi)$, $(\varphi\psi)$, $(\varphi\psi)$
  3. Nothing else is a sentence

- **Example**:
  p_1 = "Snow is white"
  q_1 = "The sky is blue"; p_1 and q_1 are atomic sentences, and, by definition of a sentence, so is "Snow is white and the sky is blue" and "Snow is white or the sky is blue."

2 | (Informal) Semantics

Dfn **2.1.** *A truth function is a function the arguments and values are T or F.*
Dfn **2.2.** *and, or correspond to*

- 'and' refers to $f_2(S_1, S_2) = \begin{cases} T & \text{for } S_1 = T, S_2 = T \\ F & \text{otherwise} \end{cases}$
- 'or' refers to $f_2(S_1, S_2) = \begin{cases} F & \text{if } S_1 = F, S_2 = F \\ T & \text{otherwise} \end{cases}$

Semantics by Visual Representation: Truth-Tables
Dfn **2.3.** *Truth table for , , , ,*

φ	ψ	φ	$(\varphi\psi)$	$(\varphi\psi)$	$(\varphi\psi)$	$(\varphi\psi)$	$\bot$
T	T	F	T	T	T	T	F
T	F		F	T	F	F	
F	T	T	F	T	T	F	
F	F		F	F	T	T	

3 | Ordinary and Formal Language

'not': 'it is not the case that …'
: ' …and …"
: "Either …or …" , "…unless …"
: if, only if, provided that, assuming
: if and only if, just in case

Some ordinary language sentences have non-standard translation

- Inclusive or: 'or' : Soup or salad comes with the meal. Accurate Translation: $P \vee S$

Some aspects of meaning are lost

- 'but','however': constrastive connotation lost
- 'moreover': connotation of additional

4 | Natural Deduction

Modus Ponens (MP)	$pq, p \,/\, q$
Modus Tollens (MT)	$pq, q \,/\, p$
Hypothetical Syll (HS)	$pq, qr \,/\, pr$
Disjunctive Syll (DS)	$pq, p \,/\, q$
Constructive Dilemma (CD)	$(pq)(rs), pr \,/\, qs$
Simplification (Simp)	$pq \,/\, p$
Conjunction (Conj)	$p, q \,/\, pq$
Addition (Add)	$p \,/\, pq$

DeMorgan's (DeM)	$(pq) :: (pq)$
	$(pq) :: (pq)$
Commutativity (Com)	$(pq) :: (qp)$
	$(pq) :: (qp)$
Associativity (Assoc)	$[p(qr)] :: [(pq)r]$
	$[p(qr)] :: [(pq)r]$
Distribution (Dist)	$[p(qr)] :: [(pq)(pr)]$
	$[p(qr)] :: [(pq)(pr)]$
Double Negation (DN)	$p :: p$
Implication (Impl)	$(pq) :: (pq)$
Transportation (Trans)	$(pq) :: (qp)$
Material Implication (Impl)	$(pq) :: (pq)$
Material Equivalence (Equiv)	$(pq) :: (pq)(qp)$
	$(pq) :: (pq)(pq)$
Exportation (Exp)	$(pq)r :: (p(qr)$

Conditional Proof (CP)

1	...	
2	$\underline{p}$	ACP
3	...	...
4	q	...
5	pq	2-4 MP

Indirect Proof (Proof by Contradiction) (IP)

1	...	
2	$\underline{p}$	ACP
3	...	...
4	qq	...
5	pq	2-4 IP